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A NUMERICAL INVESTIGATION OF THE STABILITY OF A MULTICHANNEL SYSTEM FOR CONTROLLING THE LOADING OF STRUCTURES†

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A mathematical model of test bed systems for checking the strength of aircraft structures is proposed. The transients in the control system are investigated. A proportional integro-differential regulator is tested numerically. The effectiveness of the control algorithm along a planned trajectory is demonstrated. A method of constructing a partitional matrix of the properties of the object, which increases the control quality is proposed. © 2004 Elsevier Ltd. All rights reserved.

A mathematical model of test bed systems for checking in the strength of aircraft structures is a software package which enables one to investigate the transients in an automatic control system (ACS) during the loading of an elastic aircraft structure and, also, to choose the parameters of the optimal control algorithm for specified criteria when carrying out a numerical experiment. The model can also be used as a source of additional information when making natural checks on the control beyond the state of the real object.

In the proposed mathematical model, a block ACS scheme is implemented which is represented by the following generalized structures (Fig. 1).

The program. In this block, the values of the programmed forces $\mathbf{Q}^p$ and the position $\mathbf{\Lambda}^p$ of the freely suspended aircraft are determined at any instant of time in the loading cycle with respect to a fixed system of coordinates ξ, η and ζ.

The control system. In this block, the actual time of the loading cycle is *tracked* and the control signals $\mathbf{U}$ for the slave mechanisms are generated. The programmed values of $\mathbf{Q}^p$ and $\mathbf{\Lambda}^p$ and the actual values of the forces and displacements attained up to actual instant of time constitute the input data of this block.

The loading system is an integrating module which transforms the input signal $\mathbf{U}$ into a displacement of the rods of the slave mechanisms $\mathbf{S}$.

The test unit takes account of the elastic properties of the aircraft structure and, using specified displacements $\mathbf{S}$, enables one to determine the values of the forces $\mathbf{Q}$ and the displacements. $\mathbf{\Lambda}$ which are realized for the aircraft as an absolutely rigid body.

The ACS functions in the following manner.

The load control is discretely implemented and a uniform time mesh $t_j = t_0 + i^*\Delta t$ $(i = 0, 1, \dots, n)$ is constructed for each complete loading cycle over a time from t_0 to $t_{\max}$ (the time interval for a complete cycle is T). In the actual experiments, the minimum value of the control discreteness, Δt, depends on the number of control channels, the speed of response of the apparatus used and the algorithm for calculating the control signals. At the instant of time t_i, the values of $\mathbf{Q}_i^p$, $\mathbf{\Lambda}_i^p$, $\mathbf{Q}_i$ and $\mathbf{\Lambda}_i$ are transmitted to the control unit and the control signals $\mathbf{U}_i$ are calculated using the specified algorithm. The control signals are fed to the controlling devices of the loading system and are kept constant during the discreteness interval Δt. After a time Δt, the rods of the slave mechanisms are displaced by $\Delta\mathbf{S}_i$. By the instant of time t_{j+1}, the displacements of the rods reach the values $\mathbf{S}_{i+1} = \mathbf{S}_i + \Delta\mathbf{S}_i$. The values of $\mathbf{Q}_{i+1}$ and $\mathbf{\Lambda}_{i+1}$ are calculated using the values of $\mathbf{S}_{i+1}$ and a new control step is executed.

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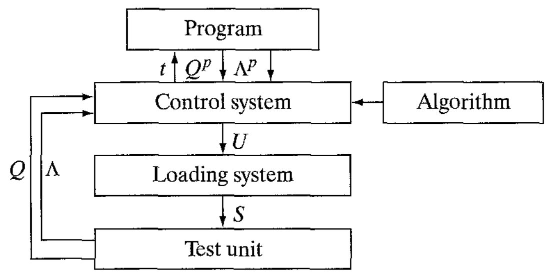

Fig. 1

To implement the loading control block scheme, it is necessary to have the equations of state of each module available. The equations of state of test bed systems for the loading of an aircraft structure with an arbitrary system of forces have been derived in [1]. In the mathematical model described below, the loading of a freely suspended structure with a system of forces parallel to the vertical axis y of the aircraft is assumed. The overall system of equations is considerably simplified in this case, which is frequently encountered in practice. We will derive this simplified system.

We will assume that, at the initial instant of time, the directions of the local (x, y- and z-axes associated with the aircraft) and the global ξ-, η- and ζ-axes are identical. Hence, the initial aircraft angles of the local system of coordinates are equal to zero: $\varphi = \psi = \theta = 0$. We will confine ourselves to the case when the angles φ and θ remain small ($\psi = 0$) during the experiments. We will express the column vector of the forces $\mathbf{Q}$ which arise during loading in terms of the dynamometer readings $\mathbf{q}$, using the matrix $\mathbf{E} = \text{diag}\{e_1, e_2, \dots, e_m\}$, when $e_j = \pm 1$ determines the directions of the action of the forces, to indicate the directions of action of the forces. Then, $\mathbf{Q} = \mathbf{Eq}$.

We will assume that the displacements of the rods of the hydraulic cylinders $\mathbf{S} = \text{col}\{s_1, s_2, \dots, s_m\}$ are positive with respect to the direction of the action of the forces. This enables us to write the condition for the combined deformation of the structure and the lever system in the form

$$(\mathbf{A}_0 + \mathbf{A}_y)\mathbf{Q} + \mathbf{M}\Lambda = \mathbf{ES}$$

Here, $\mathbf{A}_0$ and $\mathbf{A}_y$ are the compliance matrices of the lever system and the aircraft structure and $\mathbf{M}$ is a matrix which is formed using the coordinates x_j, y_j of the application of the resultant branches of the lever system on the surface of the aircraft.

The matrix $\mathbf{M}$ consists of m rows $1, -z_j, x_j$ ($j = 1, 2, \dots, m$). We note that the product of the matrix $\mathbf{M}$ and the vector $\Lambda = \text{col}\{\delta\eta, \varphi, \theta\}$ ($\delta\eta$ is the change in the position of the centre of the associated axes with respect to the global axes) gives the displacement of the points of the aircraft as an absolutely rigid body, and the product of $\mathbf{M}^T$ and $\mathbf{Q}$ determines the resultant forces and moments acting on the structure.

Replacing $\mathbf{Q}$ by the dynamometer readings $\mathbf{q}$, we obtain the system of equations

$$\mathbf{Aq} + \mathbf{EM}\Lambda = \mathbf{S}$$

where $\mathbf{A} = \mathbf{E}^T(\mathbf{A}_0 + \mathbf{A}_y)\mathbf{E}$.

In the case of a freely suspended structure, it is necessary to supplement the system with the condition for the self-balancing of all the forces $\mathbf{M}^T\mathbf{Eq} = \mathbf{g}$, where $\mathbf{g}$ is a column vector of the forces and moments of the weight of the structure in the local system of coordinates. These relations enable one to determine both the realizable forces as well as the position of the associated axes.

In order to form the complete system of equations of state of the ACS, it is necessary to take account of the functioning of the hydraulic cylinder. We will assume that a hydraulic cylinder with a valve when a control signal u_j is fed into the input circuits is described by an integrating module with a gain which depends on the load acting on the rod $\dot{s}_j = d_{\mu j}u_j$ ($j = 1, 2, \dots, m$), where

$$d_{\mu j} = \mu_j\left(1 - \frac{q_j}{q_{j\max}}\text{sign}u_j\right)^{\upsilon} \quad (1)$$

and μ_j, $q_{j\max}$ and υ are specified characteristics of the hydraulic cylinder.

For the column vector of the velocities $\dot{\mathbf{S}} = \text{col}\{\dot{s}_1, \dots, \dot{s}_m\}$, we can write

$$d\mathbf{S}/dt = \mathbf{D}_\mu\mathbf{U}$$

where $\mathbf{D}_\mu$ is a diagonal matrix with the elements (1) and $\mathbf{U}$ is the column vector of the control signals of dimension m.

For the exact specification of the gains of the hydraulic cylinder-valve system, it is necessary to use experimental data to construct an approximating function. In automatic control systems for the loading of aircraft structures, a proportional integro-differential regulator (a PID regulator) is used at the present time to generate the control signal. The control signal is calculated as the product of a certain diagonal matrix of the gains and a sum with its own factors of the proportional, differential and integral components of the deviations of the realizable loads from the programmed loads. Taking account of the resulting equations, we write the control algorithm as

$$\mathbf{U} = \mathbf{D}_\mu^{-1}\mathbf{D}_b[a_1\Delta\mathbf{q} - \alpha_2 d(\Delta\mathbf{q})/dt + \alpha_3\int\Delta\mathbf{q}dt] +$$
$$+\mathbf{D}_\mu^{-1}\mathbf{E}^T\mathbf{M}[\alpha_1\Delta\boldsymbol{\Lambda} - \alpha_2 d(\Delta\boldsymbol{\Lambda})/dt + \alpha_3\int\Delta\boldsymbol{\Lambda}dt]$$

where $\Delta\mathbf{q} = \mathbf{q}^P - \mathbf{q}$ is the error in the attainment of the programmed loads, $\mathbf{q}^P$ are the programmed loads, $\mathbf{q}$ are the realizable loads, $\mathbf{D}_b$ is the diagonal matrix of the gains, $\Delta\boldsymbol{\Lambda} = \boldsymbol{\Lambda}^P - \boldsymbol{\Lambda}$ is the error in the attainment of the programmed position of the aircraft, $\boldsymbol{\Lambda}^P$ is the programmed position of the aircraft (for example, $\boldsymbol{\Lambda}^P = 0$) ad Λ is the position of the aircraft which is realizable during the loading.

The introduction of the factor $\mathbf{D}_\mu^{-1}$ into the algorithm for calculating $\mathbf{U}$ is intended for normalizing the characteristic of the hydraulic cylinder, and the factor $\mathbf{D}_b$ enables one to take account of the elastic properties of the aircraft. The control algorithm now depends solely on the parameters α_1, α_2, α_3.

The use of this algorithm enables us not only to attain the programmed loads but also to control the position of the aircraft.

A numerical experiment to investigate such a law enabled the effect of each component to be determined and the optimal parameters to be chosen. Testing was carried out using a real aircraft structure. The following were used as the quantitative estimates:

(1) the mean error in realizing the forces p_q, which is calculated as an integral characteristic of the ratio of the modulus of the difference between the programmed and the attained forces to the maximum programmed forces in each of the m channels (as percentages) averaged over a loading cycle T

$$p_q = \frac{100}{T}\int_0^T\left[\frac{1}{m}\sum_{j=1}^{m}\frac{|q_j^P - q_j|}{q_{j\max}}\right]dt$$

(2) the mean value of the moduli of the control signals over a loading cycle divided by the maximum control stress

$$\bar{U}_{\text{mid}} = \frac{1}{T}\int_0^T\left[\frac{1}{m}\sum_{j=1}^{m}\frac{|u_j|}{U_{\max}}\right]dt$$

(3) the mean value of the moduli of the change in the control signals after a single loading step (as percentages of $U_{\max}$)

$$\Delta U = \frac{100}{nU_{\max}}\left[\sum_{i=1}^{n}\frac{1}{m}\sum_{j=1}^{m}|u_j(t_i) - u_j(t_{i-1})|\right]$$

The first parameter enables one to judge the accuracy of the clearing of the programmed forces. The second parameter indicates the accuracy of the choice of hydraulic cylinder for the loading program under investigation, and the third parameter characterizes the smoothness of the loading of the structure using of the corresponding rule for generating the control signals and the chosen step size in the discretization of the control.

In the numerical modelling of a loading cycle, the gains for each channel were specified such that the control signals calculated in terms of the pliability matrix of the aircraft – lever system and in terms of the matrix $\mathbf{D}_b$ were identical for the programmed loads

$$\mathbf{A}\mathbf{q}^P = \mathbf{D}_b\mathbf{q}^P$$

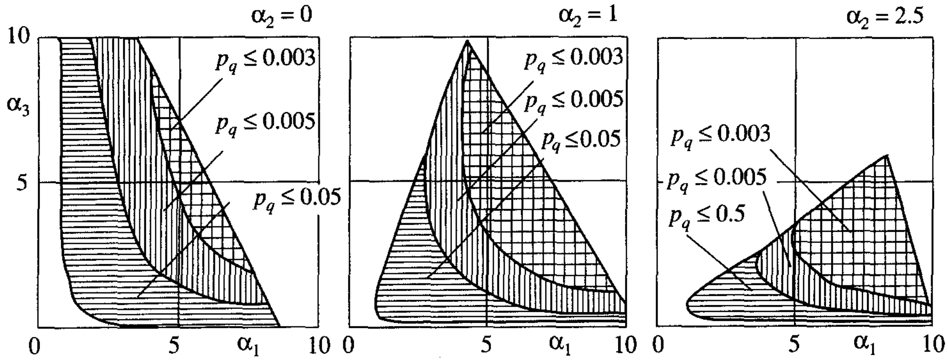

Fig. 2

Attempts to use other algorithms to calculate the gains and, in particular, optimization using gradient methods did not enable us to improve the control quality.

Numerical testing was carried out for a mode of change in the programmed forces in the form of a dashed line with a loading cycle duration of 50 s and a control discretization time interval Δt (the loading step) of 0.5, 0.05 and 0.005 s.

The results of the numerical experiment showed that a PID-regulator with a diagonal matrix of the gains can be used theoretically with a step size of 0.05 s. In this case, for the optimal choice of parameters (in a control system without any delay in the slave mechanisms), the error in the attainment of the forces does not exceed 0.15%. However, this algorithm is unstable both with respect to the parameters as well as with respect to random errors. A step size of not greater than 0.005 s is therefore needed for the control.

The domains of stability of the control with respect to the parameter α_1 and α_3 for different values of α_2 when $\Delta t = 0.005$ s are shown in Fig. 2. These domains of parameter values ensure the limiting values for the error in the load p_q shown in Fig. 2.

Testing showed that the accuracy of the loading increases and the range of values of the parameters of the PID-regulator is extended if the planned values of the loads (a control along a planned trajectory) are taken into account when generating the control signals.

The diagonal matrix $\mathbf{D}_b$, the elements of which are the normalized displacements of the rods of the hydraulic cylinders from the programmed forces, is used when implementing the control algorithm along a planned trajectory. These displacements can be determined experimentally during the static loading of a structure. Suppose, for example, that the corresponding displacements $\mathbf{S}^P$ of the hydraulic cylinder rods are determined under loading with the maximum programmed forces $\mathbf{q}^P_{\max}$. Then, the elements of $\mathbf{D}^b$ can be calculated using the formula

$$d_{bii} = s_i^P / q_{i\max}^P$$

For the displacements of the rods $\mathbf{S}^P$ from the maximum programmed forces $\mathbf{q}^P_{\max}$, we have $\mathbf{S}^P = \mathbf{D}_b\mathbf{q}^P_{\max}$. The relation holds for any programmed forces in the case of proportional loading of an aircraft. The programmed velocities of the hydraulic cylinder rods are determined as $d\mathbf{S}^P/dt = \mathbf{D}_b d\mathbf{q}^P/dt$. Such a velocity of the rods is realized by the control signals $\mathbf{U}^P = \mathbf{D}_\mu^{-1}\mathbf{D}_b\dot{\mathbf{q}}^P$. The derivatives are calculated using the "forward" difference $\dot{\mathbf{q}}^P = [\mathbf{q}^P(t + \tau_2) - \mathbf{q}^P(t)]/\tau_2$. Hence, in order to control a load without feedback to the slave mechanisms, it is necessary to supply control signals of the form

$$\mathbf{U}^P(t) = \frac{1}{\tau_2}\mathbf{D}_\mu^{-1}\mathbf{D}_b[\mathbf{q}^P(t+\tau_2) - \mathbf{q}^P(t)]$$

Taking account of the inaccuracy in determining the initial values of $\mathbf{D}_b$, we assume, in the general case, that

$$\mathbf{U}^P(t) = \alpha_4\mathbf{D}_\mu^{-1}\mathbf{D}_b[\mathbf{q}^P(t+\tau_2) - \mathbf{q}^P(t)], \quad \alpha_4 \approx 1/\tau_2$$

Diagrams of the stability domains obtained using planned values in conjunction with a PID-regulator are shown in Fig. 3. It is clear from these graphs that the domain of parameters which ensure the minimum error p_q is significantly extended.

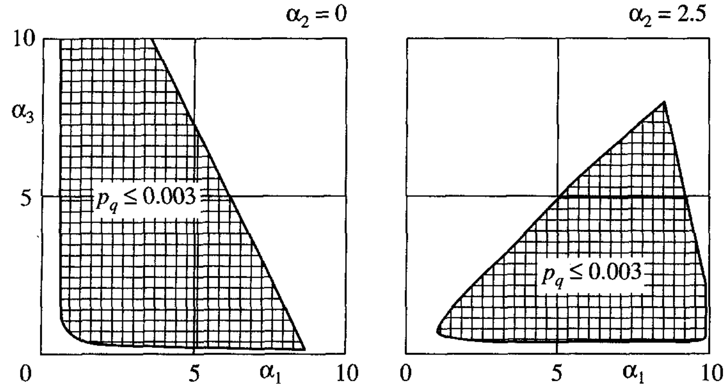

Fig. 3

The fact that the control of each channel is achieved independently of the other channels is a shortcoming of a classical PID-regulator. If a partitioned matrix $\tilde{\mathbf{A}}$, which approximately describes the properties of the structure and takes account of the mutual effects of the control channels, is specified instead of the diagonal matrix $\mathbf{D}_b$, the loading process will be more stable and the step size for the discretization of the control will be significantly larger.

We construct the control matrix $\tilde{\mathbf{A}}$ using the known values of the gains $\mathbf{D}_b$, using two matrices: a diagonal matrix $\tilde{\mathbf{A}}_0$, which approximately describes the properties of the lever system, and $\tilde{\mathbf{A}}_y$ which describes the rigidity properties of the aircraft structure in the local system of coordinates. In calculating the matrices $\tilde{\mathbf{A}}_0$ and $\tilde{\mathbf{A}}_y$, we make the following assumptions: (a) all of the elements of the diagonal matrix are equal to one another $\tilde{\alpha}_{0i}$ = const = $\tilde{\alpha}_0$; (b) the displacements of the points of the structure are solely a consequence of bending; (c) we specify the spanwise distribution of the flexural rigidity (or the distribution over the fuselage) $EJ(z) = EJ_0 e^{-kz}$.

Suppose that certain displacements of the rods are obtained due to the action of a specified programmed load in the case of the proposed change in the rigidity properties of the structure. We require that the sum of the squares of the deviations of the assumed and real displacements should be a minimum

$$H = \sum_{i=1}^{m} \left[d_{bii} q_i^P - \left(\tilde{\alpha}_0 q_i^P + e_i \sum_{j=1}^{m_w} \tilde{\alpha}_{yij} q_j^P e_j \right) \right]^2 \Rightarrow \min$$

Equating the partial derivatives of H with respect to the parameters $\tilde{\alpha}_0$, $1/EJ_0$ and k to zero, we obtain a system of non-linear equations for determining these parameters. On solving the system of non-linear equations, we calculated the pliability matrix of all the elements of the structure, treating them as beams of variable stiffness. The partitional control matrix is constructed using the pliability matrices of the wing and the fuselage which have been found.

The use of the control matrix $\tilde{\mathbf{A}}$ to generate the signals $\mathbf{U}$ enables one to take account of the reciprocal effect of the control channels. The increase in the time required to calculate $\mathbf{U}$ is a drawback, but this is compensated for by the possibility of a control with a large step size.

An algorithm using the control matrix $\tilde{\mathbf{A}}$ ensures a smaller error in implementing the loads over a wider range of variation of the parameters (Fig. 4). The domains of stability of the algorithm using a control matrix and the planned value of the signal are shown in Fig. 5.

It is seen from the graphs that the range of change in the parameters α_1 and α_3, which ensures the specified errors in the clearing of the loading program, increased more than 30-fold. Moreover, there was a qualitative change in the character of the loading at the limiting values of the parameters. The optimal parameters of a PID-regulator, which ensure the minimum error in the attainment of the forces, lie at the boundary of the stability domain. Hence, a sharp increase in the error in implementing the forces and a rapid increase in ΔU, that is, the magnitude of the change in the control signals at each loading step, is observed for parameter values slightly greater than the optimum values. When a partitioned control matrix is used, the error slowly increases when the parameters are varied in any direction from the optimal parameters.

The stability margin of the control laws using the partitioned matrix $\tilde{\mathbf{A}}$ enables one, with a loading step of 0.5 s, to obtain better loading characteristics than when using a diagonal matrix of the gains with

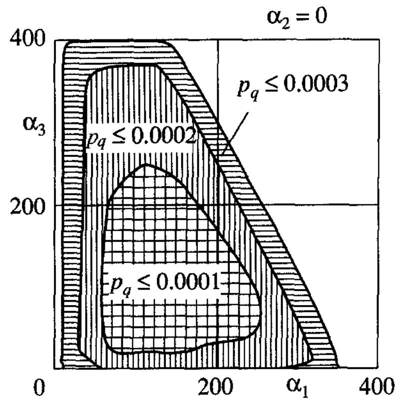

Fig. 4

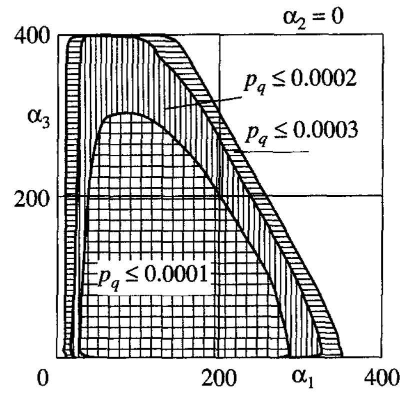

Fig. 5

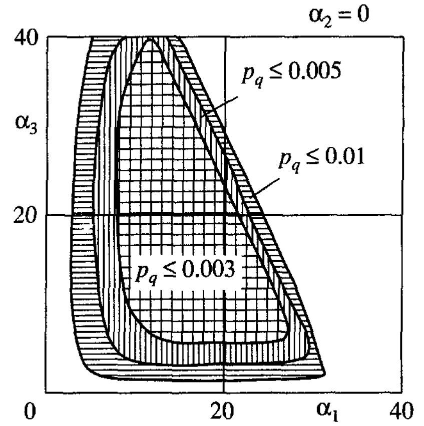

Fig. 6

a step size of 0.005 s. The stability domains, constructed for a control with a loading step size of 0.05 s using a partitioned matrix are shown in Fig. 6. If the requirements concerning the clearing of the forces are not very rigorous, load control using the matrix $\tilde{A}$ and planned values of the signals is possible even in the case of a step size of 0.5 s.

The testing of the loading control laws which was carried out enabled us to determine their stability domains with respect to the parameters and to recommend for use a proportional integral regulator with a partitioned control matrix, taking into account the planned value of the signal.

We wish to dedicate this paper to the 80th birthday of Corresponding Member of the Russian Academy of Sciences E. I. Grigolyuk.

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